Improved Method for Determining Rheological Parameters of Suspensions

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One of the problems of rheometry, which has not been satisfactorily solved until now, is measuring rheological properties of suspensions, particularly if the suspension contains settling particles. Although various methods have been used for characterization of rheological behavior of suspensions, in most cases, they appeared to be unsuitable due to such effects as phase, effective slip on the wall, and blocking of the measuring gap by particle aggregates. The instruments used included rotational coaxial-cylinder rheometers and capillary and tube rheometers. From the earlier articles by Chavan et al. (1972; Chavan and Ulbrecht, 1973) and Sawinsky et al. (1979), Kemblowski et al. (1988) developed a measuring system based on the use of a helical screw impeller, and which did not require a calibration procedure. Because of the complicated geometry of the system, however, these authors did not attempt to solve the proper set of differential equations of motion, but instead chose to perform a simplified model. Like Chavan et al. (1972), Kemblowski et al. introduced the concept of equivalent radius of the screw impeller. In order to investigate the rheological properties of settling fluid and suspensions, Connell et al. (1994) likewise modified a standard rotational rheometer by adding a recirculatory flow.

In view of these articles, the aim of this work is to develop a measuring system for determining rheological parameters of suspensions, specifically molasses containing suspended particles, based on a helical screw impeller with a draught tube. By using both Newtonian liquids of known viscosity and non-Newtonian power-law liquids, the equivalent radius of the screw impeller was evaluated and the validity of the measuring system for determining rheological parameters of suspensions was verified.

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Experimental Studies

The measuring system, which is our own design, is shown in Figure 1. The dimensions of the impeller and draught tube are summarized in Table 1. The diameter of helical screw was 0.066 m, and its length was 3–4 times its diameter. The helical screw impeller with a draught tube was located coaxially in a cylindrical vessel and was connected to a Rheotest 2.1 rheometrical unit. In order to keep the temperature con-

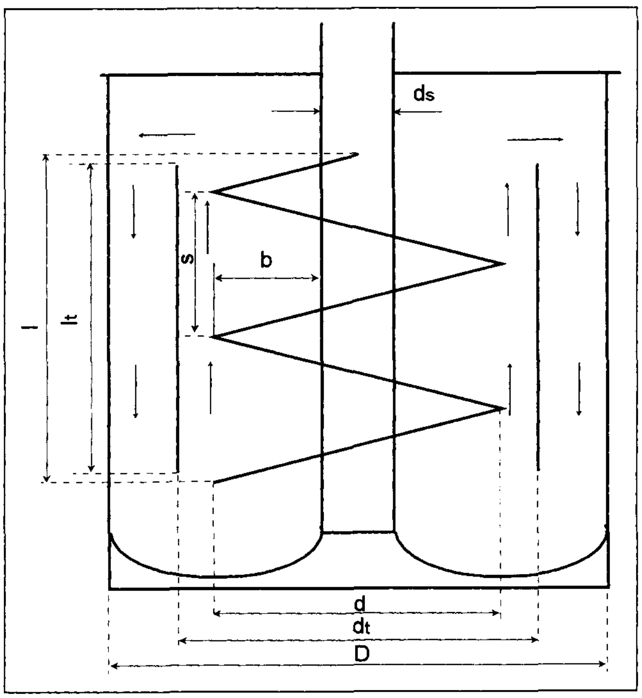

Figure 1. Geometrical variables.

Table 1. Geometrical Variables of Helical Screw Impeller with a Draught Tube

d_t/d	d_s/d	l/d	D/d	s/d	b/d	l_t/d
1.050	0.170	1.125	1.484	0.50	0.415	1.039

stant, the fluid inside the tank was heated by passing water through a jacket around the vessel.

Solutions containing 70 wt. % (at 313 K), 72.1 wt. % (at 308 K), and 75 wt. % (at 313 K) of sucrose were considered as Newtonian fluids. As non-Newtonian homogeneous fluids, a solution containing 3 wt. % of carboxymethylcellulose (CMC) at (308 K) and sugar cane molasses containing 64.1, 75.6, and 81.2 wt. % of total solids (sucrose and nonsugars) in solution (at 313 K) were studied. As suspensions, molasses with different contents of total solids in solution (64.1–81.2 wt. %) and suspended solids (20–50 wt. %) were considered. These heterogeneous systems were prepared by adding a known amount of pure sucrose crystals (density of 1.59 kg/m^3) to sugar cane molasses with different contents of sucrose in solution.

Systems were characterized by using the helical screw impeller with a draught tube. Moreover, in order to evaluate, on the one hand, the equivalent radius of the screw impeller, and verify, on the other hand, the validity of the system, the results obtained with this measuring system for the Newtonian and non-Newtonian homogeneous fluids, respectively, were compared to those obtained using S-S1, S-S2 and S-S3 cylinders of a Rheotest. Table 2 contains the results reached with the latter.

Results and Discussion

Consider the real system replaced by a model of two coaxial cylinders, where the inner cylinder has a radius equal to the equivalent radius, r_e, and the radius of the outer cylinder is, in turn, equal to that of the real screw impeller, r_t.

Assuming steady state and laminar flow, tangential and axial components of the velocity satisfy the equations (Connel et al., 1994)

$$\mu \upsilon'(r) = \alpha r + \frac{\beta}{r} \tag{1}$$

$$\mu r \omega'(r) = -\frac{M}{2\pi r^2}. \tag{2}$$

The nonslip conditions on the inner and outer cylinders yield the following boundary conditions:

$$\upsilon(r_e) = 0; \qquad \upsilon(r_t) = 0 \tag{3}$$

$$\omega(r_e) = \Omega; \qquad \omega(r_t) = 0, \tag{4}$$

where Ω is the angular velocity of the inner cylinder. The outer cylinder is fixed.

Combining Eqs. 1 and 2

$$\upsilon' = -\frac{2\pi(\alpha r^2 + \beta) r^2 \omega'}{M}. \tag{5}$$

Furthermore, if the fluid viscosity depends on the local shear rate, γ, and the fluid satisfies the Ostwald de Waele form, Eqs. 1, 2, and 5 may also be combined to give

$$\omega' = -\left(\frac{M}{2\pi m}\right)^{1/n} r^{-[1+(2/n)]} \phi^{1-(1/n)}(r, \alpha, \beta), \tag{6}$$

where

$$\phi(r,\alpha,\beta) = \left[1 + \left(\frac{4\pi^2 r^2}{M^2}\right)(\alpha r^2 + \beta)^2\right]^{-0.5}. \tag{7}$$

Integration of Eq. 6 subject to conditions 3 and 4 yields

$$\Omega = \left(\frac{M}{2\pi m}\right)^{1/n} \int_{r_e}^{r_t} r^{-[1+(2/n)]} \phi^{1-(1/n)}(r,\alpha,\beta)\, dr. \tag{8}$$

Table 2. Newtonian and Non-Newtonian Homogeneous Systems

Measuring System: Standard Rotational System						
System	T(K)	μ(Pa·)	n	m(Pa·s^n)	χ^2	Remarks
Suc. 70.0 wt. %	313	0.1141			0.0024	$\mu = 0.1148$*
Suc. 72.1 wt. %	308	0.2622			0.0014	$\mu = 0.2630$*
Suc. 75.0 wt. %	313	0.3900			0.0011	$\mu = 0.3922$*
CMC 3.0 wt. %	308		0.8751	0.2864	0.0210	
Molasses 64.1 wt. %	313		0.9800	0.0870	0.0911	81.2%**
Molasses 75.6 wt. %	313		0.9607	0.2498	0.0478	66.2%**
Molasses 81.2 wt. %	313		0.9400	2.1982	0.0316	66.7%**

Measuring System: Helical Screw Impeller with a Draught Tube							
System	T(K)	n	m(Pa·s^n)	Q(m^3/s)	λ	ϵ(%)	γ(s^{-1})
CMC 3.0 wt. %	308	0.8793	0.2853	0.449×10^{-2}	0.8489	1.63	8–154
Molasses 64.1 wt. %	313	0.9837	0.0849	0.110×10^{-1}	0.8492	1.66	15–154
Molasses 75.6 wt. %	313	0.9452	0.2316	0.525×10^{-2}	0.8491	3.81	8–154
Molasses 81.2 wt. %	313	0.9303	2.3340	0.497×10^{-3}	0.8491	3.42	1–154

*After Bates (1948).
**Sucrose in solution/solid content ratio.

Since it has been assumed steady state, the flow through the gap between the outer surface of the draught tube and the inner surface of the vessel is equal to that through the gap between the screw and the inner surface of the draught tube.

Fredrickson and Bird (1958) deduced the analytical solution of the equation of motion for the steady-state axial flow of an incompressible non-Newtonian flow in an annular space. The volume rate of flow was found to be

$$Q = 2\pi R^2 \int_k^1 v(r) q\,dq$$

$$Q = \pi R^3 \left(\frac{PR}{2m} \right)^{1/n} \int_k^1 |\lambda^2 - q^2|^{(1/n)+1} q^{-(1/n)}\,dq. \tag{9}$$

This equation can easily be integrated once λ has been determined from the following equation:

$$\int_k^l \left(\frac{l^2}{q} - q \right)^{1/n} dq = \int_l^1 \left(q - \frac{l^2}{q} \right)^{1/n} dq. \tag{10}$$

On the other hand, the flow rate through the gap between the screw and the inner surface of the draught tube (equal to that in the annular space) is given by

$$Q = 2\pi \int_{r_e}^{r_t} r v(r)\,dr, \tag{11}$$

so that, on integration by parts, it yields

$$\int_{r_e}^{r_t} r^{3-(2/n)} (\alpha r^2 + \beta) \phi^{1-(1/n)}(r, \alpha, \beta)\,dr + \left(\frac{2\pi m}{M} \right)^{1/n} \frac{MQ}{2\pi^2} = 0. \tag{12}$$

Finally, by again combining Eqs. 5 and 6, and integrating subject to Eqs. 3 and 4:

$$\int_{r_e}^{r_t} r^{1-(2/n)} (\alpha r^2 + \beta) \phi^{1-(1/n)}(r, \alpha, \beta)\,dr = 0. \tag{13}$$

The pair of nonlinear equations (Eqs. 12 and 13) determines the values of the parameters α and β, which may then be used in Eq. 8 to give the relationship between Ω and M.

To evaluate the rheological constants, m and n, the experimental data (Ω and M) obtained for each fluid were fitted by using a nonlinear regression method based on Marquardt's Algorithm (Marquardt, 1963), in which the objective function to be minimized is given by the following equation:

$$\chi^2 = \sum_{i=1}^{N} [(\Omega)_{\exp} - (\Omega)_{\mathrm{Eq.\,8}}]_i^2. \tag{14}$$

Equivalent radius determination

In order to evaluate the equivalent radius of the screw impeller, the Newtonian fluids (solutions containing 70 wt. %, 72.1 wt. % and 75 wt. % of sucrose) were considered. Thus, the rheological parameter m—viscosity—determined with a standard rotational rheometer (see Table 2) was fixed in the model described earlier. Fitting a set of 46 experimental data (Ω, M) corresponding to the three Newtonian systems yielded a value of $r_e = 0.02998$, the confidence interval and the average deviation being 0.02995–0.03000 m and 0.3%, respectively.

Cheng and Carreau (1995) proposed an equation for helical screw impellers that allows one to evaluate torque as a function of an equivalent diameter. Thus,

$$\mathrm{torque} = \frac{\pi}{2} d_t^2 l m \left(\frac{2\Omega}{n} \left(\frac{d_t}{d_e} \right)^{2/n} - 1 \right)^n. \tag{15}$$

By again fitting a set of 46 experimental data (Ω, M) corresponding to the Newtonian systems, a value of r_e equal to 0.0299 was obtained. This value is equal to that computed in this work, thus showing the good agreement observed between both procedures for determining the equivalent radius of screw helical impellers.

The equivalent radius obtained is, however, higher than that evaluated from the equation reported by Chavan et al. (1972) for a mixing system using a helical screw impeller with a draught tube:

$$\frac{d_e}{d} = \frac{d_t}{d} - \frac{\frac{2b}{d}}{\left[\ln \left(\frac{\frac{d_t}{d} - \left(1 - 2\frac{b}{d} \right)}{\frac{d_t}{d} - 1} \right) \right]}. \tag{16}$$

By substituting values of Table 1 into this equation, a value of d_e equal to 0.0502 m ($r_e = 0.0251$ m) is obtained.

Validity of proposed model

Table 2 contains the rheological parameters for all of the non-Newtonian homogeneous systems, m and n, the flow rate, Q, and the dimensionless radial coordinate, λ, calculated with the proposed model using the equivalent radius computed in the above section. As expected, the flow rate decreases when the rheological parameter m increases. On the other hand, a practically constant value of the dimensionless radial coordinate, λ, not dependent on the system under consideration is observed. The mean value of λ computed from Table 2 is 0.8491.

As can be seen, good agreement exists between the rheological parameters evaluated with the screw impeller and those calculated with the standard rotational rheometer (Table 2).

Rheological parameters of suspensions

Table 3 contains the rheological parameters for all of the suspensions (heterogeneous systems) used in the present work, m and n, the flow rate, Q, and the dimensionless radial coordinate, λ. As expected, when both the solid content and total solids in solution increase, parameter m increases, while both parameters n and Q decrease. On the other hand,

Table 3. Non-Newtonian Heterogeneous Systems*

System	n	$m(\mathrm{Pa\cdot s}^n)$	$Q(\mathrm{m}^3/\mathrm{s})$	λ	$\epsilon(\%)$	$\gamma(\mathrm{s}^{-1})$
Molasses 64.1 wt. % solids 26 wt. %	0.9371	0.2499	0.511×10^{-2}	0.8491	4.13	9–154
Molasses 75.6 wt. % solids 26 wt. %	0.9661	0.6031	0.178×10^{-2}	0.8492	4.58	4–154
Molasses 81.2 wt.% solids 20 wt. %	0.8733	6.4684	0.200×10^{-3}	0.8489	3.68	4–50
Molasses 81.2 wt. % solids 30 wt. %	0.8428	8.1760	0.170×10^{-3}	0.8488	2.34	4–50
Molasses 81.2 wt. % solids 40 wt. %	0.8598	14.24	0.870×10^{-4}	0.8488	2.24	0.3–17
Molasses 81.2 wt.% solids 50 wt. %	0.8298	37.86	0.290×10^{-4}	0.8487	2.97	0.1–10

*Measuring system: helical screw impeller with a draught tube.

as observed in the previous section, practically constant values of λ not dependent on the system are seen, the mean value being coincidental with that obtained with the non-Newtonian homogeneous systems. By fixing the mean value of λ (0.8489) in the computer program, the results of the fits reached are practically equal to those in Table 3.

In Figure 2 the experimental and predicted angular velocity obtained for two molasses with different solids contents are compared, a good agreement being observed (average relative error lower than 3%).

Conclusions

In this work a procedure based on using a helical screw impeller with a draught tube is presented. Based on the concept that the helical screw impeller has an equivalent radius, which was determined from known viscosity data of Newtonian liquids, an improved numerical procedure is presented. Comparison between the value of the impeller's equivalent radius and those computed from equations proposed in the literature shows good agreement with that by Cheng and Carreau (1995). The validity and applicability of the measuring system for determining rheological parameters of suspensions was verified by using the non-Newtonian power-law liquids.

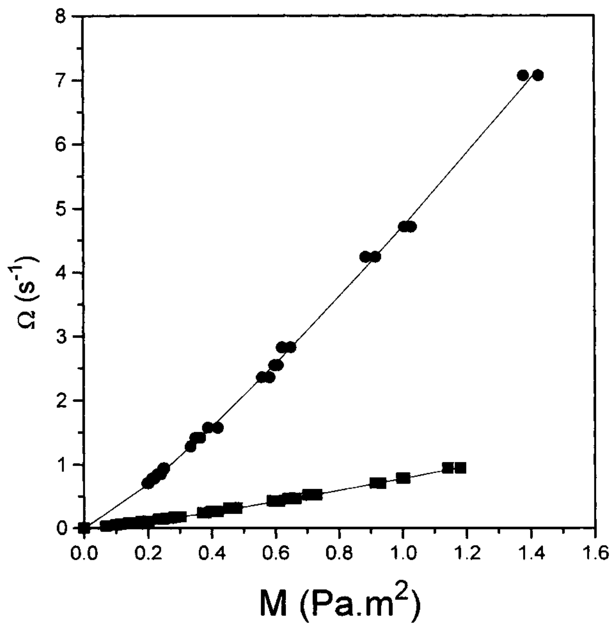

Figure 2. Experimental vs. predicted angular velocity of the inner cylinder at different values of moment per unit length (predicted values plotted as a solid line).

Systems: molasses containing 81.2 wt.% of total solids in solution and 50 wt. % of suspended solids (■); molasses containing 81.2 wt. % of total solids in solution and 30 wt. % of suspended solids (●).

Notation

b = agitator blade width, m
d = agitator diameter, m
d_e = equivalent diameter, m
d_s = shaft diameter, m
d_t = diameter of draught tube, m
D = vessel diameter, m
k = ratio of radius of inner cylinder to that of outer cylinder
l = agitator height, m
l_t = draught height, m
m = consistency coefficient, $\mathrm{Pa\cdot s}^n$
M = moment per unit length, $\mathrm{Pa\cdot m}^2$
n = flow behavior index
N = number of experiments
P = sum of forces per unit volume, Pa/m
q = dimensionless radial coordinate
r = radial coordinate, m
R = vessel radius, m
s = pitch of the screw, m
v = axial velocity, m/s

Greek letters

α = parameter in Eq. 1, Pa/m
β = parameter in Eq. 1, Pa·m
ϵ = average relative error, %
ϕ = function defined by Eq. 7
μ = viscosity, Pa·s
ω = angular velocity, rps
χ^2 = objective function defined by Eq. 14

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